## DEMOSTRACIONES

**Problema 1:** Sea $V$ un espacio vectorial real y sean $\bar{u}, \bar{v} \in V$ . Demostrar que si

$$\left\|\bar{u}+\bar{v}\right\| = \left\|\bar{u}-\bar{v}\right\|$$

entonces $\bar{u}$ y $\bar{v}$ son ortogonales.

**SOLUCIÓN:**

Demostración:

$$\left(\bar{u}+\bar{v}\middle|\bar{u}+\bar{v}\right)^{1/2} = \left(\bar{u}-\bar{v}\middle|\bar{u}-\bar{v}\right)^{1/2}$$

$$\left[\left(\bar{u}+\bar{v}\middle|\bar{u}+\bar{v}\right)^{1/2} = \left(\bar{u}-\bar{v}\middle|\bar{u}-\bar{v}\right)^{1/2}\right]^2$$

$$\left(\bar{u}+\bar{v}\middle|\bar{u}+\bar{v}\right) = \left(\bar{u}-\bar{v}\middle|\bar{u}-\bar{v}\right)$$

$$\left(\bar{u}\middle|\bar{u}+\bar{v}\right)+\left(\bar{v}\middle|\bar{u}+\bar{v}\right) = \left(\bar{u}\middle|\bar{u}-\bar{v}\right)-\left(\bar{v}\middle|\bar{u}-\bar{v}\right)$$

$$\left(\bar{u}\middle|\bar{u}\right)+\left(\bar{u}\middle|\bar{v}\right)+\left(\bar{v}\middle|\bar{u}\right)+\left(\bar{v}\middle|\bar{v}\right) = \left(\bar{u}\middle|\bar{u}\right)-\left(\bar{u}\middle|\bar{v}\right)-\left(\bar{v}\middle|\bar{u}\right)+\left(\bar{v}\middle|\bar{v}\right)$$

$$\cancel{\left(\bar{u}\middle|\bar{u}\right)}+2\left(\bar{u}\middle|\bar{v}\right)+\cancel{\left(\bar{v}\middle|\bar{v}\right)} = \cancel{\left(\bar{u}\middle|\bar{u}\right)}-2\left(\bar{u}\middle|\bar{v}\right)+\cancel{\left(\bar{v}\middle|\bar{v}\right)}$$

$$2\left(\bar{u}\middle|\bar{v}\right)+2\left(\bar{u}\middle|\bar{v}\right)=0$$

$$4\left(\bar{u}\middle|\bar{v}\right)=0$$

$\therefore \boxed{\left(\bar{u}\middle|\bar{v}\right)=0} \leftarrow$ Por tanto $\bar{u}$ y $\bar{\mathrm{v}}$ son ortogonales

**Problema 2:** Sea $V$ un espacio vectorial real y sean $\bar{u}, \bar{v} \in V$. Demostrar que:

$$\|\bar{u}+\bar{v}\|^2 + \|\bar{u}-\bar{v}\|^2 = 2\|\bar{u}\|^2 + 2\|\bar{v}\|^2$$

**SOLUCIÓN:**

Demostración:

$$\|\bar{u}+\bar{v}\|^2 + \|\bar{u}-\bar{v}\|^2 = 2\|\bar{u}\|^2 + 2\|\bar{v}\|^2$$

$$\left[\left(\bar{u}+\bar{v}|\bar{u}+\bar{v}\right)^{1/2}\right]^2 + \left[\left(\bar{u}-\bar{v}|\bar{u}-\bar{v}\right)^{1/2}\right]^2 = 2\left[\left(\bar{u}|\bar{u}\right)^{1/2}\right]^2 + 2\left[\left(\bar{v}|\bar{v}\right)^{1/2}\right]^2$$

$$\left(\bar{u}+\bar{v}|\bar{u}+\bar{v}\right) + \left(\bar{u}-\bar{v}|\bar{u}-\bar{v}\right) = 2\left(\bar{u}|\bar{u}\right) + 2\left(\bar{v}|\bar{v}\right)$$

$$\left(\bar{u}|\bar{u}+\bar{v}\right) + \left(\bar{v}|\bar{u}+\bar{v}\right) + \left(\bar{u}|\bar{u}-\bar{v}\right) - \left(\bar{v}|\bar{u}-\bar{v}\right) = 2\left(\bar{u}|\bar{u}\right) + 2\left(\bar{v}|\bar{v}\right)$$

$$\left(\bar{u}|\bar{u}\right) + \cancel{\left(\bar{u}|\bar{v}\right)} + \cancel{\left(\bar{v}|\bar{u}\right)} + \left(\bar{v}|\bar{v}\right) + \left(\bar{u}|\bar{u}\right) - \cancel{\left(\bar{u}|\bar{v}\right)} - \cancel{\left(\bar{v}|\bar{u}\right)} + \left(\bar{v}|\bar{v}\right) = 2\left(\bar{u}|\bar{u}\right) + 2\left(\bar{v}|\bar{v}\right)$$

$\therefore$ $\boxed{2\left(\bar{u}|\bar{u}\right) + 2\left(\bar{v}|\bar{v}\right) = 2\left(\bar{u}|\bar{u}\right) + 2\left(\bar{v}|\bar{v}\right)}$ $\leftarrow$ Queda demostrada la igualdad