$$(x + xy^2)dx + (y - x^2y)dy = 0$$

Input:

$$(x + x\,y^2)\,dx + (y - x^2\,y)\,dy = 0$$

ODE names:

Separable equation

$$-\frac{y'(x)}{\frac{1+y(x)^2}{y(x)}} = \frac{x}{1 - x^2}$$

Bernoulli's equation

$$y'(x) = \frac{x\,y(x)}{-1 + x^2} + \frac{x}{(-1 + x^2)\,y(x)}$$

ODE classification:

first-order nonlinear ordinary differential equation

Differential equation solutions:

$$y(x) = -\sqrt{e^{2c_1}\,x^2 - e^{2c_1} - 1}$$

$$y(x) = \sqrt{e^{2c_1}\,x^2 - e^{2c_1} - 1}$$

Mostrar pasos

$(x+xy^2)dx+(y-x^2y)dy=0$: $y=\sqrt{e^{2c_1}x^2-e^{2c_1}-1}$, $y=-\sqrt{e^{2c_1}x^2-e^{2c_1}-1}$

Pasos

$$(x+xy^2)dx+(y-x^2y)dy=0$$

Ecuación diferencial ordinaria de primer orden de variables separables *Ocultar definición*

Una EDO de primer orden de variables separables tiene la forma $N(y)\cdot y'=M(x)$

Sea y la variable dependiente. Dividir entre dx:

$$x+xy^2+(y-x^2y)\frac{dy}{dx}=0$$

Sustituir $\frac{dy}{dx}$ con y'

$$x+xy^2+(y-x^2y)y'=0$$

Reescribir como una EDO de primer orden de variables separables *Mostrar pasos*

$$\frac{y}{y^2+1}y'=\frac{x}{(x+1)(x-1)}$$

$N(y)\cdot y'=M(x)$

$N(y)=\frac{y}{y^2+1}$, $M(x)=\frac{x}{(x+1)(x-1)}$

Resolver $\frac{y}{y^2+1}y'=\frac{x}{(x+1)(x-1)}$: $\frac{1}{2}\ln(y^2+1)=\frac{1}{2}\ln((x+1)(x-1))+c_1$ *Mostrar pasos*

Despejar y: $y=\sqrt{e^{2c_1}x^2-e^{2c_1}-1}$, $y=-\sqrt{e^{2c_1}x^2-e^{2c_1}-1}$ *Mostrar pasos*

$y=\sqrt{e^{2c_1}x^2-e^{2c_1}-1}$, $y=-\sqrt{e^{2c_1}x^2-e^{2c_1}-1}$

$$(y^2 - xy - x^2)dx + x^2 dy = 0$$

(y^2-xy-x^2)dx+x^2dy=0

Extended Keyboard Upload

Input:

$$(y^2 - x\,y - x^2)\,dx + x^2\,dy = 0$$

ODE names:

Riccati's equation

$$y'(x) = -\frac{y(x)^2}{x^2} + \frac{y(x)}{x} + 1$$

Homogeneous equation

$$y'(x) = 1 + \frac{y(x)}{x} - \left(\frac{y(x)}{x}\right)^2$$

ODE classification:

first-order nonlinear ordinary differential equation

Differential equation solution:

$$y(x) = \frac{x\left(x^2 - e^{2c_1}\right)}{e^{2c_1} + x^2}$$

$(y^2 - xy - x^2)dx + x^2 dy = 0$: $\frac{\ln\left|\frac{y}{x}+1\right|}{2} - \frac{\ln\left|\frac{y}{x}-1\right|}{2} = \ln(x) + c_1$

Pasos

$(y^2 - xy - x^2)dx + x^2 dy = 0$

Sea y la variable dependiente. Dividir entre dx:

$y^2 - xy - x^2 + x^2 \frac{dy}{dx} = 0$

Sustituir $\frac{dy}{dx}$ con y'

$y^2 - xy - x^2 + x^2 y' = 0$

Despejar y': $y' = \frac{x^2 + xy - y^2}{x^2}$

Substitute $y = ux$, u is a function of x

$(ux)' = \frac{x^2 + xux - (ux)^2}{x^2}$

Simplificar

$(ux)' = 1 + u - u^2$

$(ux)' = xu' + u$

$xu' + u = 1 + u - u^2$

$xu' + u = 1 + u - u^2$: $\frac{\ln|u+1|}{2} - \frac{\ln|u-1|}{2} = \ln(x) + c_1$

Sustituir en la ecuación $u = \frac{y}{x}$

$\frac{\ln\left|\frac{y}{x}+1\right|}{2} - \frac{\ln\left|\frac{y}{x}-1\right|}{2} = \ln(x) + c_1$

$$(2xe^y + y^2e^x + 2x)dx + (x^2e^y + 2ye^x)dy = 0$$

(2xe^y+y^2e^x+2x)dx+(x^2e^y+2ye^x)dy=0

Extended Keyboard Upload

Input:

$$(2xe^y + y^2e^x + 2x)dx + (x^2e^y + 2ye^x)dy = 0$$

Exact equation:

$$\left(2x + 2e^{y(x)}x + e^x y(x)^2\right)dx + \left(e^{y(x)}x^2 + 2e^x y(x)\right)dy = 0$$

ODE classification:

first-order nonlinear ordinary differential equation

Differential equation solution:

$$x^2\left(e^{y(x)} + 1\right) + e^x y(x)^2 = c_1$$

Mostrar pasos

$$\left(2xe^y + y^2e^x + 2x\right)dx + \left(x^2e^y + 2ye^x\right)dy = 0: \quad x^2e^y + e^xy^2 + x^2 = c_1$$

Pasos

$$\left(2xe^y + y^2e^x + 2x\right)dx + \left(x^2e^y + 2ye^x\right)dy = 0$$

Ecuación diferencial exacta *Ocultar definici*

Una EDO de la forma $M(x,y) + N(x,y)y' = 0$
es una ecuación diferencial exacta si se cumple lo siguiente:

1. Si existe una función $\Psi(x,y)$ tal que $\Psi_x(x,y) = M(x,y), \quad \Psi_y(x,y) = N(x,y)$
2. $\Psi(x,y)$ tiene derivadas parciales continuas: $\frac{\delta M(x,y)}{\delta y} = \frac{\delta^2\Psi(x,y)}{\delta y \delta x} = \frac{\delta^2\Psi(x,y)}{\delta x \delta y} = \frac{\delta N(x,y)}{\delta x}$

Sea y la variable dependiente. Dividir entre dx:

$$2xe^y + y^2e^x + 2x + \left(x^2e^y + 2ye^x\right)\frac{dy}{dx} = 0$$

Sustituir $\frac{dy}{dx}$ con y'

$$2xe^y + y^2e^x + 2x + \left(x^2e^y + 2ye^x\right)y' = 0$$

La ecuación tiene la forma de una ecuación diferencial exacta: $M(x,y) + N(x,y)y' = 0$

$$\Psi_x(x,y) = M(x,y) = 2xe^y + y^2e^x + 2x, \quad \Psi_y(x,y) = N(x,y) = x^2e^y + 2ye^x$$

Si las condiciones se cumplen, entonces $\Psi_x + \Psi_y \cdot y' = \frac{d\Psi(x,y)}{dx} = 0$

La solución general es $\Psi(x,y) = C$

Verificar que $\frac{\delta M(x,y)}{\delta y} = \frac{\delta N(x,y)}{\delta x}$: Verdadero *Mostrar pas*

Encontrar $\Psi(x,y)$: $\Psi(x,y) = x^2e^y + e^xy^2 + x^2 + c_1$ *Mostrar pas*

$$\Psi(x,y) = c_2$$

$$x^2e^y + e^xy^2 + x^2 + c_1 = c_2$$

Combinar las constantes

$$x^2e^y + e^xy^2 + x^2 = c_1$$

$$(xy^2 + y)dx + (x\ln x)dy = 0$$

(xy^2+y)dx-(xlnx)dy=0

Extended Keyboard Upload

Input:

$$(x y^2 + y)\, dx - (x \log(x))\, dy = 0$$

Bernoulli's equation:

$$y'(x) = \frac{y(x)}{x \log(x)} + \frac{y(x)^2}{\log(x)}$$

ODE classification:

first-order nonlinear ordinary differential equation

Differential equation solution:

$$y(x) = -\frac{\log(x)}{c_1 + x}$$

$(xy^2 + y)dx - (x\ln(x))dy = 0$: $\quad y = \frac{\ln(x)}{-x + c_1}$

Pasos

$(xy^2 + y)dx - (x\ln(x))dy = 0$

Ecuación diferencial de Bernoulli

La ecuación diferencial de Bernoulli tiene la forma $y' + p(x)y = g(x)y^n$

Sea y la variable dependiente. Dividir entre dx:

$$xy^2 + y - x\ln(x)\frac{dy}{dx} = 0$$

Sustituir $\frac{dy}{dx}$ con y'

$$xy^2 + y - x\ln(x)y' = 0$$

Reescribir como una ecuación diferencial de Bernoulli

$$y' - \frac{1}{x\ln(x)}y = \frac{1}{\ln(x)}y^2$$

$y' + p(x)y = q(x)y^n$

$p(x) = -\frac{1}{x\ln(x)}, \quad q(x) = \frac{1}{\ln(x)}, \quad n = 2$

La solución general se obtiene sustituyendo $v = y^{1-n}$ y resolviendo $\frac{1}{1-n}v$

Transformar a $\frac{1}{1-n}v' + p(x)v = q(x)$: $\quad -v' - \frac{v}{x\ln(x)} = \frac{1}{\ln(x)}$

Resolver $-v' - \frac{v}{x\ln(x)} = \frac{1}{\ln(x)}$: $\quad v = \frac{-x + c_1}{\ln(x)}$

Sustituir en la ecuación $v = y^{-1}$: $\quad y^{-1} = \frac{-x + c_1}{\ln(x)}$

Despejar y: $\quad y = \frac{\ln(x)}{-x + c_1}$

$$y = \frac{\ln(x)}{-x + c_1}$$

$$(x+1)^2y' + 3(x+1)y = 2$$

(x+1)^2y'+3(x+1)y=2

Extended Keyboard Upload

Input:

$$(x+1)^2\, y'(x) + 3\,(x+1)\, y(x) = 2$$

ODE classification:

first-order linear ordinary differential equation

Alternate forms:

$$(x+1)\left((x+1)\, y'(x) + 3\, y(x)\right) = 2$$

$$(x+1)\left(x\, y'(x) + y'(x) + 3\, y(x)\right) = 2$$

$$\left(x^2 + 2\,x + 1\right) y'(x) = (-3\,x - 3)\, y(x) + 2$$

Expanded form:

$$x^2\, y'(x) + 2\,x\, y'(x) + y'(x) + 3\,x\, y(x) + 3\, y(x) = 2$$

Differential equation solution:

$$y(x) = \frac{c_1}{(x+1)^3} + \frac{x^2}{(x+1)^3} + \frac{2\,x}{(x+1)^3}$$

Mostrar pasos

$$(x+1)^2y' + 3(x+1)y = 2: \quad y = \frac{x^2+2x+c_1}{x^3+3x^2+3x+1}$$

Pasos

$$(x+1)^2y' + 3(x+1)y = 2$$

Ecuación diferencial ordinaria lineal de primer orden *Ocultar definici*

Una EDO lineal de primer orden tiene la forma $y'(x) + p(x)y = q(x)$

Reescribir como una EDO lineal de primer orden *Mostrar pas*

$$y' + \frac{3}{x+1}y = \frac{2}{(x+1)^2}$$

$$y'(x) + p(x)\cdot y = q(x)$$

$$p(x) = \frac{3}{x+1}, \quad q(x) = \frac{2}{(x+1)^2}$$

Hallar el factor de integración: $\mu(x) = x^3 + 3x^2 + 3x + 1$ *Mostrar pas*

Mostrar pas

Escribir la ecuación con la forma $(\mu(x)\cdot y)' = \mu(x)\cdot q(x)$: $\left(\left(x^3+3x^2+3x+1\right)y\right)' = 2(x+1)$

Resolver $\left(\left(x^3+3x^2+3x+1\right)y\right)' = 2(x+1)$: $y = \frac{x^2+2x+c_1}{x^3+3x^2+3x+1}$ *Mostrar pas*

$$y = \frac{x^2+2x+c_1}{x^3+3x^2+3x+1}$$